

WARM UP

Simple Interest

Basic Formulas	Extra Formulas
$I = Prt$ $A = P + I$	$A = P + Prt$ $A = P(1 + rt)$

1. For the formulas above, write what each letter represents.
 - I =
 - P =
 - r =
 - t =
 - A =

2. I invest \$3500 at 2.5% simple interest for 2 years.
 - a. How much interest do I earn?

 - b. How much do I end up with?

3. A 10 year T-note has a face (ending) value of \$3000. It earns 4% interest. What is the purchase price?

Compound Interest $A = P \left(1 + \frac{r}{n}\right)^{nt}$	same formula, different letters: version in your book: $P_N = P_0 \left(1 + \frac{r}{k}\right)^{k \cdot N}$
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4. For the formulas above, write what each letter represents.
 - A or P_N =

 - P or P_0 =

 - r =

 - n or k =

 - t or N =

5. I invest \$3500 at 2.5% compounded daily for 2 years.
- How much do I end up with?
 - How much interest do I earn?

Tuesday Annuities	Thursday: Payout Annuities Loans Remaining Loan Balance Review Sheet for Test 1
Tuesday Which Equation to Use Solving For Time	Thursday Test 1 on Problem Solving & Finance

Annuities (page 204)

An annuity is an investment instrument that allows you to contribute money at regular intervals, and the fund guarantees a fixed interest rate. The money accumulates to an ending balance which will be available at some point in the future.

Annuity Formula

$$P_N = \frac{d((1+\frac{r}{k})^{N \cdot k} - 1)}{(\frac{r}{k})}$$

P_N is the balance in the account after N years

d is the regular deposit amount

r is the annual interest rate in decimal form

k is the number of compounding periods in one year

If the compounding frequency isn't explicitly stated, assume that the compounding frequency is the same as the number of deposits made in a year.

6. My new year's resolution is to set aside \$100 each month into a retirement account. I found a fund that guarantees 6% interest annually.
 - a. If I can stick to this plan faithfully, how much will I have in 20 years? (Answer: \$46,204.09)

 - b. How much would I have contributed to the fund in 20 years? (Answer: \$24,000)

 - c. How much interest would I have earned? (Answer: \$22,204.09)

7. I realize that I really need to have \$200,000 saved up in 20 years. What would I need to deposit each month in order to make that happen? Assume I still am earning 6% annual interest rate. (Answer: \$432.86)

8. [Try it now 2] A more conservative investment account pays 3% interest. If you deposit \$5 a day into this account,
 - a. how much will you have after 10 years? (Answer: \$21,282.07)
 - b. How much of that money did you contribute? (Answer: \$18,250)
 - c. How much of that money is from interest? (Answer: \$3,032.07)(detailed solution on page 220)

Payout Annuity (Thursday)

With a payout annuity, you start with an amount of money in an account, and then withdraw money at regular intervals. After each withdrawal, the remaining balance continues to earn interest. The account will eventually be used up.

Payout Annuity Formula

$$P_0 = \frac{d(1 - (1 + \frac{r}{k})^{-N \cdot k})}{(\frac{r}{k})}$$

P_0 is the balance in the account at the beginning

d is the regular withdrawal amount

r is the annual interest rate in decimal form

k is the number of compounding periods in one year

N is the number of years we plan to take withdrawals

Brief summary:

Compound Interest - one deposit

Annuity - many deposits

Payout Annuity - many withdrawals

9. I want to set myself up to be able to withdraw \$1300 a month from a retirement account for a total of 30 years. The account offers 8% interest. How much money will I need to have saved up by the time I retire? (Answer: \$177,168.54) ([online calculator](#) for present value of an annuity)
10. I ended up having \$200,000 saved up by the time I retired. The account still has 8% interest, and I plan to make withdrawals for 30 years. What monthly amount will I be able to withdraw? (Answer: \$1467.53)
11. [Try It Now 3] A donor gives \$100,000 to a university, and specifies that it be used to give annual scholarships for the next 20 years. If the university can earn 4% interest, how much can they give in scholarships each year? (Answer: \$7358.18 see page 220 for detailed solution)
12. I hope to retire in 20 years. How much do I need to deposit monthly into an account earning 8% annually in order to save 177,168.54? Hint: Use the Annuity Formula. (Answer: \$300.79) ([online calculator](#) for the future value of an annuity)

Loans

WARM UP

1. I have a lump sum of \$100,000. I want to withdraw monthly payments for the next 10 years, and I found an account that pays 9% annually. What amount can I withdraw each month?
2. If I plan to deposit \$200 per month into an account earning 5% annually for the next 30 years, how much will I end up with?

Answers:

1. \$1266.76
2. \$166,451.73

Loans

The loans we are learning about include: Auto Loans, Mortgages, Conventional Loans, Installment Loans, Amortized Loans. They will all be calculated the same way. (Loans we will not be covering include payday loans, add-on loans, and other types of loans.)

When you borrow an amount to buy a house or a car, you essentially receive a lump sum, and then repay the loan with regular payments. During the repayment period, you are also paying interest on the balance of the loan. The balance decreases over time, but the interest works against you since it increases the balance.

A loan to you is a Payout Annuity to the lender. They “invest” a lump sum, then receive regular payments. They end up getting back more than they loaned you, because of the interest.

We do not have a new formula. The Payout Annuity Formula is the formula we'll be using here.

Payout Annuity Formula/Loans Formula

$$P_0 = \frac{d(1 - (1 + \frac{r}{k})^{-N \cdot k})}{(\frac{r}{k})}$$

P_0 is the balance in the account at the beginning (the amount of the loan)

d is the loan payment amount (monthly payment, annual payment, etc)

r is the annual interest rate in decimal form

k is the number of compounding periods in one year

N is the length of the loan in years

The loan formula assumes you make regular payments, and that you are paying interest on the balance of the loan.

Loans Skills:

- Start with a monthly payment amount, an interest rate, a compounding frequency, and the number of years of your loan. Figure out what loan amount you can afford.
 - Or start with a loan amount, interest rate, compounding frequency, number of years, and generate the payment amount.
 - Find the remaining loan balance partway through a loan period
3. You create a budget, and decide you can afford a \$200 monthly car payment. An auto loan from your credit union charges 3% interest for 60 months (5 years).
 - a. What is the most expensive car you can afford to buy? (Answer: \$11,130.47)
 - b. What is the total amount of all your payments? (Answer: \$12,000)
 - c. What is the total amount of the interest on the loan? (Answer: \$869.53)
 4. A friend wants to buy a car, but plans to pay it off in 3 years instead of 5 years. They find a car that costs \$10,000. The credit union offers 36 months loans at 4%. What will the car payment be? (Answer: \$295.24)

Remaining Loan Balance (Omit)

- ~~5. Let's say that your friend takes out the 3 year loan. A year later, they want to know what the remaining balance on their loan is.~~

~~This is how to re-frame the question: What loan amount now would be paid off with 2 years of the \$295.24 monthly payments?~~

~~Answer: \$6798.86~~

- ~~6. A couple purchases a home with a \$180,000 mortgage at 4% for 30 years with monthly payments.
 - a. What is their monthly payment? (you need this to answer the following question)
 - b. What will the remaining balance be after 5 years?~~

~~Answer:~~

- ~~a. \$859.35 (book answer on page 215 differs slightly because they are rounding a little during the problem, instead of just at the end)~~
- ~~b. \$162,805.99 is the balance after 5 years. (detailed solution on page 216)~~